

Mathematics Standard 2 Year 12

Financial Mathematics Topic Guidance

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[Topic focus 3](#_Toc496190579)

[Terminology 3](#_Toc496190580)

[Use of technology 3](#_Toc496190581)

[Background information 3](#_Toc496190582)

[General comments 4](#_Toc496190583)

[Future study 4](#_Toc496190584)

[Subtopics 4](#_Toc496190585)

[MS-F4: Investments and Loans Paperclip icon 5](#_Toc496190586)

[Subtopic focus 5](#_Toc496190587)

[F4.1: Investments 5](#_Toc496190588)

[Considerations and teaching strategies 5](#_Toc496190589)

[Suggested applications and exemplar questions 5](#_Toc496190590)

[F4.2: Depreciation and loans 6](#_Toc496190591)

[Considerations and teaching strategies 6](#_Toc496190592)

[Suggested applications and exemplar questions 7](#_Toc496190593)

[MS-F5: Annuities Paperclip icon 8](#_Toc496190594)

[Subtopic focus 8](#_Toc496190595)

[Considerations and teaching strategies 8](#_Toc496190596)

[Suggested applications and exemplar questions 8](#_Toc496190597)

# Topic focus

*Financial Mathematics* involves the application of knowledge, skills and understanding of number to earning, spending, investing, saving and borrowing money.

Knowledge of financial mathematics enables students to analyse different financial situations, to calculate the best options for given circumstances, and to solve financial problems.

The study of financial mathematics is important in developing students’ ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources effectively.

# Terminology

|  |  |  |
| --- | --- | --- |
| account balance  annual fee  annual interest rate  annuity  appreciated value  asset  compound interest  compounding period  contribution amount  credit  credit card  credit card statement  daily interest rate | declining-balance method depreciation  dividend  dividend yield  exponential modelling  fee  franked dividend  future value  future value interest factors ****  inflation  interest-free period  interest payable  interest rate | linear modelling  minimum payment  monthly repayment  percentage annual interest rate  portfolio  present value  recurrence relation  repayment  reducing balance loan  shares  simple interest  term  withdrawal |

# Use of technology

Spreadsheets are an ideal tool for demonstrating the difference between simple interest and compound interest, and particularly the importance of the cumulative compounding effect.

Graphing software and spreadsheets can be used to calculate, graph and analyse loan repayments, particularly in the case of reducing-balance loans.

Students can use spreadsheets or other appropriate software to construct tables and graphs to illustrate annuities.

Students should be able to create a spreadsheet to simulate a credit card statement, including the calculation of the interest payable and the minimum payment due.

The websites of financial institutions are a source of interactive applications and software that can be used to investigate concepts of financial mathematics.

# Background information

Compound interest is a fundamental part of investing and borrowing money. The concept of compounding may be used in developing an understanding of depreciation, and also the growth of populations and the depletion of natural resources.

# General comments

Students require access to current information from a range of sources including, but not limited to, newspapers, journals, magazines, real bills and receipts, and the internet.

Teachers and students should also have access to actual financial information and products, for example, examples of credit card statements should be used in learning and teaching.

Students should be familiar with correct terminology in the topic areas and be able to use this terminology when justifying or explaining their solutions to problems.

Students should have a strong understanding of how the compound interest formula can be used to calculate depreciation by substituting a negative value for .

Calculations of annuities should be performed using a table of interest factors. Calculation of annuities through the use of a table rather than through the use of formulae, provides for the development of an appropriate understanding of the underlying concepts.

This topic has strong links to the A4.2 Non-Linear Relationships topic and it would be valuable to discuss these concepts explicitly with students.

# Future study

In this topic, students model and solve practical problems and learn about financial concepts that have an important role in areas beyond secondary school. They will be provided with opportunities to become familiar with the terminology used in financial matters and gain many of the financial literacy skills that are vital for them to competently manage their finances in the future.

# Subtopics

* MS-F4: Investments and Loans Paperclip icon
* MS-F5: Annuities Paperclip icon

## MS-F4: Investments and Loans Paperclip icon

### Subtopic focus

The principal focus of this subtopic is to calculate and compare the value of different types of investments, including shares, over a period of time and to gain an understanding of reducing balance loans and that an asset may depreciate rather than appreciate in value over time.

Students develop awareness of mechanisms to optimise their financial position, both now and into the future, justifying their thinking and reasoning mathematically.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

## F4.1: Investments

### Considerations and teaching strategies

* The ‘compounded’ value of a dollar is also known as the ‘future value’ of a dollar.
* In the financial world, the compound interest formula is known as the future value formula and is expressed as: .
* The single amount that needs to be invested when the future value is known can be calculated using the formula: .
* Calculations of simple interest and compound interest should include interest rates expressed other than as ‘per annum’.
* Tables of compounded values of $1, such as the table below, can be used to calculate the amount to which money invested in a compound interest account has grown in a given period.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Compounded values of $1 | | | | |
| Interest rate per period | | | | |
| Periods | 1% | 5% | 10% | 15% | 20% |
| 1 | 1.010 | 1.050 | 1.100 | 1.150 | 1.200 |
| 2 | 1.020 | 1.103 | 1.210 | 1.323 | 1.440 |
| 3 | 1.030 | 1.158 | 1.331 | 1.521 | 1.728 |
| 4 | 1.041 | 1.216 | 1.461 | 1.750 | 2.074 |

### Suggested applications and exemplar questions

* An amount of $3000 is invested and compounded annually at 5%. Use the table of compounded values of $1 above to find the value of the investment after three years.

Solution: Value of investment after three years

* Grandparents wish to save $10 000 for their grandchild’s university expenses, and to have this amount available in eight years’ time. Calculate the single sum they need to invest at 5% pa compounded annually.
* Determine the single sum to be deposited if $10 000 is required in five years’ time and a rate of 3% pa, compounded quarterly, is available.
* A principal of $1000 is invested for three years at an interest rate of 5.6% pa compounded half-yearly. Determine how much needs to be invested to achieve the same interest if the interest rate was 3.5% pa compounded monthly.
* It is predicted that a particular painting will appreciate at a rate of 5% per annum. Calculate its predicted value in 2020 if it was purchased in 2010 for $48 000.
* An investor has 2000 shares. The current share price is $1.50 per share. The investor is paid a dividend of $0.30 per share.

1. What is the current value of the shares?
2. Calculate the dividend yield.

## F4.2: Depreciation and loans

### Considerations and teaching strategies

* Current interest rates for various lending institutions should be compared.
* Car loans are personal loans with a fixed monthly repayment. These loans are reducing balance loans and may be paid off in less time than the original term by increasing the monthly repayments, or by making additional payments.
* Students use hand-held calculators to create a table of loan repayments for the first few repayments of a loan.
* Given a row in a table of loan repayments, students generate the next row using a hand-held calculator.
* Students use digital technology such as a spreadsheet to calculate the next row, or a particular value for a row/column, in a table of loan repayments, for example, calculate the next row (ie when = 5) in the loan repayment table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Loan table | | | | |
|  | Amount = | $50 000 | This table assumes the same number of days in each month, ie | |
|  | Annual interest rate = | 10% |
|  | Monthly repayment () = | $600 |  | |
|  | Principal () | Interest () |  |  |
| 1 | $50 000 | $416.67 | $50 416.67 | $49 816.67 |
| 2 | $49 816.67 | $415.14 | $50 231.81 | $49 631.81 |
| 3 | $49 631.81 | $413.60 | $50 045.40 | $49 445.40 |
| 4 | $49 445.40 | $412.05 | $49 857.45 | $49 257.45 |

* Students calculate the depreciation of an asset using the compound interest formula with a negative value for . They compare the effect of this declining-balance method with the straight-line method they used in Year 11.
* Students should have access to credit card statements issued by major Australian financial institutions. Details that could identify an individual need to be changed sufficiently, or deleted, to protect privacy. These details include, but are not limited to, names, account numbers, and addresses.
* Students should access suitable websites that provide information on credit cards and related calculations.
* The calculation of interest and other credit card charges varies between the different issuers of credit cards. Interest is typically charged for retail purchases, cash advances, balance transfers and the amount still owing from the previous month. In this course, the same interest rate will be assumed to apply for all transaction types, and students will be required to calculate interest for amounts still owing from the previous month, cash advances and retail purchases, but not for balance transfers.
* For interest calculations on credit cards, compound interest is to be assumed. It is also to be assumed that interest is calculated on the daily outstanding account balance for each transaction and is applied at the end of the statement period. The daily interest rate is used for this calculation.
* Note: The daily interest rate is the annual percentage rate divided by 365, for example, an annual rate of 16.5% is equivalent to a daily rate of 0.000452 (expressed as a decimal to three significant figures).
* Students should calculate the interest charged, closing balance, and minimum payment due given the annual interest rate, opening balance, at least two purchase transactions (item, date, amount), one cash advance (date, amount) and one or more repayments (date, amount).

### Suggested applications and exemplar questions

* Plan a spreadsheet for a reducing-balance loan using paper, pen and calculator, and then construct the spreadsheet. Consider car loans, travel loans, loans for capital items, home loans, as well as other types of loans.
* Students use a loan spreadsheet or an online simulator to vary the amount borrowed, the interest rate, and the repayment amount. They determine the answers to suitable ‘what if’ questions, eg What is the effect on the term of the loan and the amount of interest paid if:

1. there is an interest rate rise
2. more than the minimum monthly repayment is paid
3. there is an interest rate rise in the case where more than the minimum monthly repayment is already being paid?

* Use a prepared graph of ‘amount outstanding’ versus ‘repayment period’ to determine when a particular loan will be half-paid.
* Calculate and compare the amount of depreciation of motor vehicles for different ages of the vehicles. Note: The depreciation in the first year of a new car can exceed 35%. For many vehicles, depreciation levels out to between 7% and 10% per annum after the first three years.
* Students create a credit card statement using a spreadsheet.
* Students compare, by making calculations, the costs associated with credit cards from different lenders. This should include consideration of the interest rates offered and fees.
* Students can use a spreadsheet or other appropriate digital technology to create a graphical representation comparing the use of two credit cards with different interest rates for the same purchases.
* Credit card payment calculations should involve fees and interest-free periods.

## MS-F5: Annuities Paperclip icon

### Subtopic focus

The principal focus of this subtopic is the nature and mathematics of annuities, the processes by which they accrue, and ways of optimising their value as an investment.

Students develop awareness of the use of annuities in their lives, for example superannuation and home loans.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

### Considerations and teaching strategies

* Students can calculate the values of annuities using formulae.
* Teachers may wish to construct a table of future value interest factors for annuities and a table of present value interest factors for annuities using the appropriate formulae:

and where is the contribution per period paid at the end of the period, is the interest rate per compounding period, and is the number of periods.

### Suggested applications and exemplar questions

* Students calculate the future value of an annuity () using a table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table of future value interest factors | | | | | |
|  | Interest rate per period | | | | |
| Period | 1% | 2% | 3% | 4% | 5% |
| 1 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | 2.0100 | 2.0200 | 2.0300 | 2.0400 | 2.0500 |
| 3 | 3.0301 | 3.0604 | 3.0909 | 3.1216 | 3.1525 |

For example, using the table, the future value of an annuity of $1200 per year for three years at 5% pa is .

* Students use a table to calculate the present value of an annuity () or to calculate the contribution per period, for example to determine the monthly repayment in the case of a reducing‑balance car loan.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table of present value interest factors | | | | | | |
|  | 0.0060 | 0.0065 | 0.0070 | **0.0075** | 0.0080 | 0.0085 |
|  |  |  |  |  |  |  |
| 45 | 39.33406 | 38.90738 | 38.48712 | 38.07318 | 37.66545 | 37.26383 |
| 46 | 40.09350 | 39.64965 | 39.21263 | 38.78231 | 38.35859 | 37.94133 |
| 47 | 40.84841 | 40.38714 | 39.93310 | 39.48617 | 39.04622 | 38.61311 |
| **48** | 41.59882 | 41.11986 | 40.64856 | **40.18478** | 39.72839 | 39.27924 |
| 49 | 42.34475 | 41.84785 | 41.35905 | 40.87820 | 40.40515 | 39.93975 |
| 50 | 43.08623 | 42.57113 | 42.06459 | 41.56645 | 41.07653 | 40.59470 |

In the example indicated in the table, the present value interest factor for a loan with interest rate 0.0075 per month, expressed as a decimal, (or 9% pa) over 48 months (four years) is 40.18478.

The monthly repayment for a car loan of $8000 at 9% pa for four years is calculated as follows:

to the nearest dollar.