# Sample Unit – Mathematics Standard 1 – Year 12

***Sample for implementation for Year 12 from Term 4, 2018***

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| **Unit Title** | Modelling with Graphs | **Duration** | 20–24 hours |
| **Strand** | Algebra | **Topic** | MS-A3: Types of Relationships |
| **Subtopic focus**The principal focus of this subtopic is the graphing and interpretation of relationships, and the use of simultaneous linear equations in solving practical problems. Students develop their ability to communicate concisely, use equations to describe and solve practical problems, and use algebraic or graphical representations of relationships to predict future outcomes.Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students. | **Resources**Using graphs to solve simultaneous equations: <http://www.transum.org/software/SW/Starter_of_the_day/Students/Using_Graphs.asp>Filling containers with water: <https://teacher.desmos.com/waterline/walkthrough#Tumbler>Air quality data: <http://www.environment.nsw.gov.au/aqms/hourlydata.htm>Graphing stories: <http://graphingstories.com/>Slot car game <http://splash.abc.net.au/home#!/media/1827634/slotcars-game>App: Action Grapher in the Apple App Store (a free app)Graphing games: <http://theuniverseandmore.com/>Slideshows: <http://www.projectmaths.ie/workshops/workshop4/UnderstandingGraphs.pdf> and<http://www.johnogilvie.s-lanark.sch.uk/maths/powerpoints/graphs_of_real_life_situations.ppt>Depth-time graphs: <https://www.geogebra.org/m/S4Yc2fda>Population projections: <http://www.planning.nsw.gov.au/projections> and<https://www.google.com.au/publicdata/explore?ds=d5bncppjof8f9_&met_y=sp_pop_totl&idim=country:AUS:CAN&hl=en&dl=en#!ctype=l&strail=false&bcs=d&nselm=h&met_y=sp_pop_totl&scale_y=lin&ind_y=false&rdim=region&ifdim=region&tdim=true&hl=en_US&dl=en&ind=false>Population growth: <https://www.google.com.au/publicdata/explore?ds=d5bncppjof8f9_&met_y=sp_pop_grow&idim=country:AUS:USA:JPN&hl=en&dl=en>Generate information and graphs on topics of choice: <http://www.wolframalpha.com/> |

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| **Outcomes** | **Assessment Strategies** |
| A student:* uses algebraic and graphical techniques to evaluate and construct arguments in a range of familiar and unfamiliar contexts MS1-12-1
* represents the relationships between changing quantities in algebraic and graphical forms MS1-12-6
* chooses and uses appropriate technology effectively and recognises appropriate times for such use MS1-12-9
* uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to others

MS1-12-10 | Informal Assessment:At the start of the unit the teacher assesses students’ prior learning using a variety of different strategies, including:* students working in small groups to brainstorm what they already know about linear graphs
* a mastery quiz
* class discussion to develop a mind map of prior learning.

During the unit the teacher may assess student progress using strategies including:* observing student engagement during in-class problem-solving tasks
* monitoring the completion of homework tasks
* collecting samples of student work to informally assess individual progress
* providing opportunities for students to contribute to class discussion and/or group work
* posing key questions when working in one-to-one situations with students.
* starting each lesson with a brief (5 min) quiz that reviews the key concepts of previous lessons and key skills that will be required in the lesson that will follow.

Formal Assessment:An investigative task in which practical situations such as the number of apps downloaded from the Apple store are modelled using spreadsheets and graphing technology and analysed. |
| This unit of work builds on the Year 11 subtopic MS-A2 Linear Relationships. Teachers should differentiate the learning experiences to meet the needs of the students in their class. Teachers may decide that for particular groups of students they:• comprehensively review related Year 11 content before studying the Year 12 content• briefly review the related Year 11 content.Although this unit of work has been written to address the content of both A3.1: Simultaneous linear equations and A3.2: Graphs of practical situations teachers may wish to teach these as two separate units. |

| **Content** | **Teaching, learning and assessment strategies** |
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| **A3.1: Simultaneous linear equations**Students:* solve a pair of simultaneous linear equations graphically, by finding the point of intersection between two straight-line graphs, using technology **Paperclip icon**  Information and communication technology capability icon
* develop a pair of simultaneous linear equations to model a practical situation **AAM Paperclip icon** Critical and creative thinking icon  Information and communication technology capability icon
* solve practical problems that involve finding the point of intersection of two straight-line graphs, for example determine and interpret the break-even point of a simple business problem where cost and revenue are represented by linear equations **AAM** **Paperclip icon** Work and enterprise
 | Introductory Activity: The teacher introduces the topic by posing a problem such as: Joe and Jen are setting up a coffee cart in the city. Set-up costs were $250 and on average, it costs them $1 to produce a cup of coffee. They decide to sell coffee for $4 a cup. How many cups do they need to sell to break even?The class discusses possible ways of solving the problem which may include the strategies of guess and check, setting up a table of values or drawing a graph. The teacher may need to define the term ’break-even’. Students add this term to their personal glossary of terms.Guided Practice: The teacher reviews the skill of graphing linear equations both by hand from a table of values and using graphing software. Independent Practice: Students practise constructing the graph of linear equations and check their graphs using graphing software.The teacher defines the simultaneous solution of a pair of linear equations as a point which is common to both graphs. This definition is linked to the concept of the break-even point. Independent Practice: Students practise finding the break-even point or simultaneous solution of pairs of linear equations* given the graphs of both lines
* given one graph and then constructing the second on the same set of axes
* using graphing software to graph two linear equations.

Class Discussion: Will there always be a simultaneous solution for a pair of linear equations? Why or why not? Guided Practice: The teacher reviews the technique of developing an equation from information presented as words. For example write an equation that illustrates the relationship between:* the income generated when cupcakes are sold for $3.50 each
* the cost of hiring an electrician if there is a $80 call out fee and the charges are then $60 per half hour or part thereof
* the cost of running an incandescent light globe if the in-rush of electricity when it is initially turned on costs 1 cent and it costs 4 cents per hour to run.

Class Discussion:For the previous examples, if the relationship was graphed as a straight line what limitations could the model have? Are there any parts of the graph that have no practical meaning?Independent Practice: Students practise recognising the related quantities and selecting a variable to represent them. They then write an equation that translates the words into algebra. Guided Practice: The class returns to the introductory problem. With teacher assistance as necessary, students determine the variables in the problem, graph the linear equations they have written and solve the problem.Class Discussion:* The two lines divide the number plane into four regions. What is the practical interpretation of each of these zones?
* What is the practical meaning of the *y‑*intercept of both of the lines?

Independent Practice: Students solve practical problems using simultaneous equations. For example:A student needs a new printer and after research, has narrowed the decision to two options. One option is to purchase a new laser printer for $275. It will print 2000 pages per cartridge and cartridges cost $28. Alternatively an inkjet printer costs $47 and prints 500 pages per cartridge and cartridges cost $30. Which option should the student buy? Justify your answer.Other applications include choosing an energy-efficient appliance or car; distance/time problems; mixing proportions of ingredients to make a blend; event organising; problems of supply and demand.Challenge:How many grams of 70% dark chocolate and 20% milk chocolate are needed to be mixed together to produce 1 kg of 40% chocolate?Issue the formal assessment task: Discuss the nature and scope of the task, the marking rubric and the way in which the students can access feedback and support. Provide students with an opportunity to ask questions about the task and seek clarification if necessary. Clarify procedures for submission of digital files that may be included with the task. |
| **A3.2: Graphs of practical situations**Students:* construct a graph from a table of values both with and without technology  Information and communication technology capability icon
	+ use values of physical phenomena, eg the growth of algae in a pond over time, or the rise and fall of the tide against a harbour wall over time to plot graphs and make predictions
* sketch the shape of a graph from a description of a situation, for example the time passed and the depth of water in different shaped containers, or the speed of a race car as it moves around different shaped tracks Critical and creative thinking icon
* determine the best model (linear or exponential) to approximate a graph by considering its shape, using technology where appropriate **AAM** **Paperclip icon** Critical and creative thinking icon  Information and communication technology capability icon
* identify the strengths and limitations of linear and non-linear models in given practical contexts **AAM** Critical and creative thinking icon
 | Introductory Activity: The teacher provides students with a variety of different graphs that illustrate practical situations. For example tide charts, share prices, weather or house prices. These should be in different forms, both printed and digital. Working in pairs or small groups, students examine at least two of the graphs and respond to some stimulus questions. For example:* Describe the information that the graph is illustrating.
* What is the purpose of the graph?
* Can a prediction be made using the graph?
* Why might the information be presented as a graph and not a table?
* Are there any interesting or different features of the graph that are worthy of notice?
* Is the type of graph a suitable choice for the information displayed? Why or why not? If not, what type of graph would have been better?
* Are there any questions that arise as a result of reading the information in the graph?

The class regroups. Each graph is displayed to the class. Members of the groups that explored the displayed graph report the results of their conversations to the class.Practical Activity:The teacher provides a number of containers or bottles of different shapes. Water is poured steadily into a container and students observe the height of the water level. Students are asked to draw a graph using time on the horizontal scale and height on the vertical scale. The activity is repeated so that students have the opportunity to confirm their graph. The class then discusses the graphs the students have produced, acknowledging that the graphs are not accurate. This activity is repeated for other types of containers.Virtual Activities:* The activity above can be repeated using technology. For example <https://www.geogebra.org/m/S4Yc2fda> or <https://teacher.desmos.com/waterline>.
* Students can explore the graph produced when a car travels around a race track using the Action Grapher App or <http://splash.abc.net.au/home#!/media/1827634/slotcars-game>.
* Students watch a video of a practical situation and then draw a graph to illustrate the information. For example <http://graphingstories.com/>.

Practical Activity:Students are each given a card with a brief description of a journey of approximately 30 m. For example: * Walk briskly to the 10 m mark, stand still for 5 seconds, then walk very slowly to the 30 m mark
* Start walking fast but as you get farther along slow the speed at which you are walking.

A number line is marked on the ground and each student acts out their journey on the number line while the rest of the class draws the displacement-time graph that illustrates the journey. The class regroups to review the graphs they have produced.Virtual Activity:Students simulate a moving man using <https://phet.colorado.edu/en/simulation/moving-man>. Students should experiment with the man for a short time. The teacher can then draw a distance–time graph on the board and the students move the man appropriately in order to recreate the graph.Card Sort Game:Students are given a set of cards that include a graph of a practical situation and descriptions of situations. For each graph, there are three possible descriptions. Working in pairs, the students decide on the description that matches the graph. Once they have eliminated the pairs of matching graph and description, they draw the graphs for the remaining descriptions.Formal Assessment Reminder:The teacher reminds students of the formal assessment task and provides opportunities for students to seek assistance.Exploring Step Graphs:Students can explore step graphs using the activity resource provided. (Refer to Sample 1 Resource: Step Graphs). In class discussion, students explore the graph features, such as why the graph is made up from horizontal lines or what the closed and empty circles mean. Students read information from the graph.The teacher explains the difference between a linear model and a non-linear model, using examples drawn from the graphs students have already encountered in the topic. The teacher then explains the specific differences between a linear model and an exponential model using, for example the simple and compound interest graphs the students have encountered in the Year 11 Financial Mathematics topic. The main differences between the two types of graphs are discussed.Population Growth Simulation:Students construct a simple spreadsheet that simulates the growth of a population. For example:A certain type of algae grows at a rate of 5% each day. When first observed, the algae covered 8 m2 of the surface of a pond. Draw a graph that illustrates the growth of the algae over the next two weeks. Students describe the graph and use it to:* predict the amount of algae after a specific number of days
* explore the limitations of the model
* make changes to the rate of growth and observe the changes in the graph that results.

Students use technology to explore a variety of different graphs from sources such as <https://www.google.com.au/publicdata/directory> or <http://tides.willyweather.com.au/nsw/sydney.html> or <http://www.wolframalpha.com>. They describe the graphs as linear or non-linear models and select those which illustrate exponential relationships.Formal Assessment Submission:The teacher may provide in-class time for students to finalise their response to the formal assessment task and assist with the submission of any digital files they may include. |

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| **Prior knowledge** | **Questions and prompts for Working Mathematically** | **Summary of technology opportunities** |
| Experience in: * constructing simple spreadsheets
* translating relationships expressed in words into mathematical symbols
* using graphing software to graph linear functions.
 | What is the same and different about …?Is … an example of …?Are there any more examples of …?Explain why …How can we be sure that …?Of what is this a special case …?Sort or organise the following according to …What else can be concluded?What if …? | Graphing software should be used to create graphs and investigate the similarities and differences between a variety of linear relationships.Use a spreadsheet to generate tables of values and then draw the associated graphs. Use graphing software to find the point of intersection of lines.Explore graphs of physical phenomena using data and graphs available on the internet. |
| Reflection on learning and evaluation – to be completed by teacher during or after teaching the unit. |