Mathematics sample unit	Order of Operations	Stage 3

Scope and sequence summary	Duration: 2 weeks
Substrand: S3 NA Multiplication and Division 2 (part)	Detail: 8 activities

Outcomes	Key considerations	Overview
〉 describes and represents mathematical situations in a variety of ways using mathematical terminology and some conventions MA3-1WM 〉 selects and applies appropriate problem-solving strategies, including the use of digital technologies, in undertaking investigations MA3-2WM 〉 gives a valid reason for supporting one possible solution over another MA3-3WM 〉 selects and applies appropriate strategies for multiplication and division, and applies the order of operations to calculations involving more than one operation MA3-6WM	*Key ideas* • Recognise and use grouping symbols • Apply the order of operations in calculations *Language* Students should be able to communicate using the following language: equals, **operations**, **order of operations**, **grouping symbols**, **brackets**, **number sentence**, is the same as. 'Grouping symbols' is a collective term used to describe brackets [ ], parentheses ( ) and braces { }. The term 'brackets' is often used in place of 'parentheses'. Often in mathematics, when grouping symbols have one level of nesting, the inner pair is parentheses ( ) and the outer pair is brackets [ ], eg $360 \div [4 \times (20 - 11)]$. *Background information* An 'operation' is a mathematical process. The four basic operations are addition, subtraction, multiplication and division. Other operations include raising a number to a power and taking a root of a number. An 'operator' is a symbol that indicates the type of operation, eg +, –, × and ÷.	*This unit of work encompasses:* • some of the content of S3 Multiplication and Division *Links to learning across the curriculum* Students will practise their **literacy** skills as they read and interpret descriptions of real-life contexts in order to obtain the mathematical information necessary for the required calculation. Students will employ **critical and creative thinking** as they develop knowledge, skills and understanding of the order of operations in mathematics and explain why completed number sentences are or are not correct. Students will explore the functionality of a variety of calculators or calculator applications to determine whether or not these **information and communication technologies** apply the order of operations automatically to obtain a result.

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Explore the use of brackets and the order of operations to write number sentences (ACMNA134) • use the term 'operations' to describe collectively the processes of addition, subtraction, multiplication and division • investigate and establish the order of operations using real-life contexts, eg I buy six goldfish costing $10 each and two water plants costing $4 each. What is the total cost?'; this can be represented by the number sentence $6 \times 10 + 2 \times 4$ but, to obtain the total cost, multiplication must be performed before addition ▸ write number sentences to represent real-life situations (Communicating, Problem Solving)	***Activity 1: Shopping List*** Students use a real-life situation to establish the need for an agreed convention to govern the order in which mathematical operations must be performed when there is more than one operation. • Discuss the notion of an 'operation' in the mathematical sense as a process of combining numbers. Use the term 'operations' to describe those operations already known to students, ie addition, subtraction, multiplication and division. Students should be made aware that in future years of their mathematics study they will be introduced to other operations, such as 'squaring' a number. • As a class, students create a shopping list of items for a class celebration, choosing items advertised in supermarket catalogues and/or takeaway food menus. For the activity to work, students will need more than one of some items. • Discuss ways in which the total cost of the shopping list could be calculated, then form small groups to calculate the total cost. If necessary, students can be prompted to consider each of the operations and how they may (or may not) be used to find the total cost. • Groups create and write down possible number sentences to calculate the total cost. Students should be encouraged to write a few words to explain each part of their number sentences and keep track of what has already been calculated. Each group should create at least two different number sentences for the calculation. • A member of each group writes their number sentences on the board. As a class, discuss the similarities and differences between each group's number sentences. It should be possible to identify two basic approaches: – the use of addition to add each item individually – the use of multiplication to calculate the cost of multiple items and then addition to obtain the total of each part. • The teacher summarises these approaches for comparison on the board by writing a single number sentence for each, working from left to right across the board, eg:	*Resources* • Supermarket catalogues and/or takeaway food menus, OR supermarket and/or restaurant websites

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	– addition only: $5 + $3 + $3 + $3 + $3 + $6 + $6 + $10 + $10 + $8 – with multiplication: $5 + 4 × $3 + 2 × $6 + 2 × $10 + $8 • Discuss how to go about using these number sentences to calculate the correct total cost, and how the numbers may be misinterpreted to give an incorrect total cost. Possible prompts include: – How can you calculate the correct total cost using the first number sentence? (Responses could include: 'Just add them up', 'Add from left to right' and 'Add in any order'.) – How can you calculate the correct total cost using the second number sentence? (Responses could include: 'You have to do the multiplication first' and 'Work out all the multiplications and then add them up'.) – What happens if you just work from left to right in each number sentence? Will you get the same total cost? • As a class, determine that there is a need for an agreed convention to govern the order in which mathematical calculations are performed when there is more than one operation. Introduce the 'order of operations' as the convention used for the order in which operations are to be performed. Conclude that the class has just discovered one of the rules of the order of operations, ie that multiplication is performed before addition.	
	Activity 2: Can You Crack the 'Order of Operations' Puzzle? Students work in pairs or small groups with a list of number sentences, each of which shows the correct answer. These number sentences should be carefully selected to allow students to 'discover' the order of operations for number sentences involving the operations of addition, subtraction, multiplication and division. In particular, they should illustrate that multiplication and division need to be performed before addition and subtraction, and that when the question involves two or more operations of multiplication and division, these operations should be performed working from left to right. When a question involves two or more operations of addition and subtraction, these should also be performed working from left to right.	*Scootle resources* • L6543: Exploring order of operations (Levels 1 and 4 are appropriate prior to learning the role of parentheses) • M009636: Order of operations (teaching strategies)

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	• Students examine the number sentences provided and write a description (in words, using appropriate mathematical terminology) of the order in which each operation must be performed to obtain the correct answer. Teachers should provide a model of the type of description required, eg for $15 + 2 \times 10 - 30 = 5$, students write 'First, multiply 2 by 10 to get 20. Then add 15 and 20 to get 35. Then subtract 30 from 35 to get 5.' This provides an ideal opportunity to revise and reinforce correct mathematical terminology. • With teacher guidance, including appropriate prompts, students generalise what they have observed and develop the order of operations for number sentences involving addition, subtraction, multiplication and division. Possible prompts include: – What happens when the question involves an addition or subtraction and a multiplication? – Do you always perform multiplication before division? – What happens when there are only two operations, and both of them are division? Does it matter which division you do first? • Students record the order of operations that they have generated for questions involving addition, subtraction, multiplication and division, eg 'First, do multiplications and divisions, working from left to right. Then do additions and subtractions, working from left to right.' • Students practise the order of operations by evaluating expressions that each involve two or more operations of addition, subtraction, multiplication and division.	

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• recognise that the grouping symbols ( ) and [ ] are used in number sentences to indicate operations that must be performed first • recognise that if more than one pair of grouping symbols are used, the operation within the innermost grouping symbols is performed first • perform calculations involving grouping symbols without the use of digital technologies, eg $5 + (2 \times 3) = 5 + 6 = 11$ $(2 + 3) \times (16 - 9) = 5 \times 7 = 35$ $3 + [20 \div (9 - 5)] = 3 + [20 \div 4] = 3 + 5 = 8$	***Activity 3: Let's Group It!*** Students use a real-life situation to establish the need for a way to indicate that in some situations, addition (or subtraction) is to be performed before multiplication (or division) in a number sentence. • As a class, students consider a real-life situation where addition should be performed before multiplication, and consider how the situation can be represented using a number sentence, eg: – Potato chips come in variety boxes that contain 4 packets of 'sour cream and chive' flavour, 3 packets of 'sweet chilli' flavour and 3 packets of 'original' flavour. Jay buys 6 variety boxes. How many packets of chips does he have in total? • Students suggest possible strategies to solve the real-life situation. The teacher records these strategies on the board. Teachers may need to prompt students to consider an approach where addition is performed first and discuss the difficulty in writing that as a single number sentence that will result in the correct answer, since the convention for the order of operations means that multiplication would be performed before addition. Using the previous example about potato chips, the answer can be obtained by multiplying the number of each flavour by 6, and then adding, ie using the expression $4 \times 6 + 3 \times 6 + 3 \times 6$, for which the correct order of operations means that multiplication is performed before addition in order to obtain the correct answer. The answer can also be obtained by adding the number of packets in one box first, and then multiplying by 6. Students would need to calculate the result of the expression $4 + 3 + 3$ first, and then multiply by 6. Students should consider and discuss why the number sentence $4 + 3 + 3 \times 6$ is not an expression that will result in the correct answer. • Introduce the notion of 'grouping symbols' as a way of indicating that a particular operation is to be performed first and demonstrate their use by writing a single expression using ( ) that will result in the correct answer for the real-life situation. Using the previous example about potato chips, the correct answer can be obtained using the single expression $(4 + 3 + 3) \times 6$, as the grouping symbols group the items to be added and indicate that the operations in brackets must be performed first. • Use the term 'grouping symbols' to describe collectively	*Scootle resources* • L6543: Exploring order of operations (Levels 2 and 3 use grouping symbols)

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	parentheses ( ), brackets [ ] and braces { }. Recognise that the word 'brackets' is often used instead of 'parentheses' for ( ), and that the term 'square brackets' is often used when referring to [ ]. • With teacher guidance, students discuss and record the place of grouping symbols in the order of operations, eg 'First, do operations in grouping symbols/brackets. Second, do multiplications and divisions, working from left to right. Then do additions and subtractions, working from left to right.' Some students may benefit from being shown a mnemonic device to assist them in remembering the order of operations. • With teacher guidance, students consider expressions involving more than one pair of grouping symbols and determine that in such situations, the operation within the innermost pair of grouping symbols is performed first, eg for $20 \times [16 - (5 + 7)]$, calculate 5 + 7 first, then subtract this result from 16, then multiply by 20. • Students practise the order of operations by performing calculations that each involve at least one pair of grouping symbols.	
• apply the order of operations to perform calculations involving mixed operations and grouping symbols, without the use of digital technologies, eg $32 + 2 - 4 = 34 - 4 = 30$ addition and subtraction only, therefore work from left to right $32 \div 2 \times 4 = 16 \times 4 = 64$ multiplication and division only, therefore work from left to right $32 \div (2 \times 4) = 32 \div 8 = 4$ perform operation in grouping symbols first $(32 + 2) \times 4 = 34 \times 4 = 136$ perform operation in grouping symbols first $32 + 2 \times 4 = 32 + 8 = 40$ perform multiplication before addition ✸ ▸ investigate whether different digital technologies apply the order of operations (Reasoning)	***Activity 4: How to 'Operate' in Mathematics*** • Students work in small groups to create a guide describing the order of operations in order to instruct others on how to 'operate' in mathematics. The guide may take a variety of forms, such as a poster, multimedia presentation, song or poem. • Students consolidate their knowledge, skills and understanding of the order of operations by evaluating a wide variety of expressions in appropriate worksheets and/or games.	*Scootle resources* • L6543: Exploring order of operations (Levels 1 to 4) *Other online resources* • Online games, eg – 'Numbers Game Solver' based on the SBS TV program *Letters and Numbers* – in order to practise the order of operations, instruct students to write a single number sentence to obtain the target – Exploring Order of Operations − Use It

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	Activity 4: How to 'Operate' in Mathematics • Students work in small groups to create a guide describing the order of operations in order to instruct others on how to 'operate' in mathematics. The guide may take a variety of forms, such as a poster, multimedia presentation, song or poem. • Students consolidate their knowledge, skills and understanding of the order of operations by evaluating a wide variety of expressions in appropriate worksheets and/or games.	from LearnAlberta Interactives – Super Maths World, then select 'Number' and 'BODMAS' *Extension* • The 'Four Fours' problem: Students express each of the integers from 1 to 20 using only the digits 4, 4, 4 and 4 and the operations +, −, × and ÷, grouping symbols and decimal points, eg 1 = 44 ÷ 44 2 = 4 ÷ 4 + 4 ÷ 4 13 = (4 − .4) ÷ .4 + 4. This activity can be extended further to include all integers from 1 to 100 if it is believed that students can manage the use of square roots, powers and factorials, eg 97 = 4 × 4! + 4 ÷ 4. Solutions to the Four Fours problems are readily available online, eg www.mathsisfun.com.
	Activity 5: Jump to It! • Use chalk to write a series of expressions on concrete in the playground. Expressions should include all four operations and grouping symbols. Write the answer to each expression on a card that is placed facedown on the concrete beside the expression. • Students work in pairs or small groups taking turns within the pair or group to calculate the expression by (physically) jumping	*Resources* • Chalk • Cards on which to write answers to the expressions

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	to the relevant operation of the expression in the order required to achieve the correct answer. Another member of the pair or group checks the answer on the card. Once the correct answer is obtained, the pair or group moves to the next expression.	
	Activity 6: Do Calculators Use the Order of Operations? • Review the order of operations used to correctly evaluate number sentences with more than one operation and/or grouping symbols. • The teacher explains that calculators (either handheld or on a computer, mobile phone or other device) are each programmed to work in a particular way, and that some are not programmed to apply the accepted order of operations, eg if you enter $5 + 4 \times 2$, some calculators will simply work from left to right (ie do the addition before the multiplication) and therefore give the incorrect answer. So, you can't trust every calculator to get the order of operations correct! • The teacher explains that the class will test a variety of calculators to see if they use the order of operations by designing a set of number sentences. The teacher models an example and explains which part of the order of operations it is designed to test, eg 'To test that multiplication is performed before addition, we need a question that will identify a calculator that simply works from left to right. Such calculators would do addition before multiplication and therefore give an incorrect answer. However, if a calculator we test uses the order of operations, it will perform multiplication before addition and give us the correct answer. So a number sentence to test is $4 + 8 \times 2 = 20$.' • Working in small groups, students create a set of number sentences to test whether various calculators use the order or operations, or just complete the operations in the order that they are entered (ie from left to right). • Students use their number sentences to test a variety of calculators to see whether or not they use the order of operations.	*Resources* • A variety of calculators, including: – standard school calculators – calculator tools on computers – calculator tools on mobile phones or tablets – online calculators – basic large-key calculators (this type of calculator typically does not use the order of operations)

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• recognise when grouping symbols are not necessary, eg 32 + (2 × 4) has the same answer as 32 + 2 × 4	***Activity 7: Brackets or No Brackets*** *Part 1* • Students are presented with a variety of completed number sentences involving mixed operations and at least one set of grouping symbols. Some of the grouping symbols included should be grouping symbols that are actually not necessary. • As each number sentence is shown, the teacher points at each set of grouping symbols and asks the class if they are necessary in order to obtain the answer. Students use actions to respond, such as holding up their arms as if to show biceps when grouping symbols are necessary, and holding their arms crossed in front of their bodies when grouping symbols are not necessary. *Part 2* • Students are presented with a variety of number sentences involving mixed operations but no grouping symbols. Some of the number sentences are correct, but some require one or more sets of grouping symbols in order to make them correct. • Students are required to place the minimum number of pairs of grouping symbols in the given number sentences to make incorrect number sentences into correct number sentences.	
	Activity 8: Match the Word Problem • Students are provided with a set of 12 cards that consist of six word problems (all requiring more than one operation to find the solution) and six number sentences that can be used to calculate the solution to the six word problems. The word problems involve the same numbers combined using different operations to prevent students from guessing by simply matching numbers from the word problem instead of thinking about the operations required. A possible set of word problems and related number sentences is: – Slavek wants to buy a pair of jeans marked $60 and a shirt marked $40. At the cash register, the attendant tells him that the store is having a 'pay only half' sale. How much will Slavek need to pay? (60 + 40) ÷ 2 – For a complete school uniform, Alex needs a pair of trousers and a shirt. A pair of trousers costs $60 and a shirt costs	*Resources* • Matching activity consisting of six word problems and six related number sentences. Each word problem uses the same whole numbers and requires more than one operation to find the solution. *Extension* • Students can complete the same activity with more complex expressions, decimal

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	\$40. Alex buys two complete school uniforms. How much will she need to pay? $(60 + 40) \times 2$ – At the school clothing shop, trousers are sold for \$60 and shirts are sold for \$40. In February, the shop sold 20 pairs of trousers and 30 shirts to new students. What was the total amount of money that the clothing shop received? $60 \times 20 + 40 \times 30$ – Jing-Wei wants to buy a jacket marked \$60 and two pairs of jeans each marked \$40. How much will she need to pay? $60 + 40 \times 2$ – Rasheed wants to buy a pair of jeans marked \$60 and a shirt marked \$40. At the cash register, the attendant tells him that the store is having a 'half-price shirts' sale. How much will he need to pay? $60 + 40 \div 2$ – Each school in the region ordered 20 'Year 6' jerseys that cost \$60 each. The delivery charge for each school was \$40. There are 30 schools in the region. What was the total cost of the jerseys for the region? $(60 \times 20 + 40) \times 30$ • Students work in small groups to match the number sentences to the word problems. • Students explain, verbally or in writing, how each component of the number sentence matches the related word problem.	values and/or where there is more than one correct expression that represents each word problem, and/or where there are some expressions that do not match any of the given word problems.

Assessment overview

- Students calculate the value of each of a number of expressions involving more than one operation and/or pair of grouping symbols.
- Students are given a range of completed number sentences. They are asked to determine if these are correct, and to explain why or why not, eg 'A student calculated the answer to $3 + 4 \times 8$ to be 56. Is the student correct? Explain why or why not.'
- Students select ten whole numbers between 1 and 100. They express each of the numbers using four other numbers and at least three different operations in a number sentence, eg $65 = 3 \times 22 - 4 \div 4$.
- Students insert grouping symbols, but only where necessary, to make number sentences correct, eg $9 + 3 - 2 \times 8 = 17$.