# Sample Unit – Mathematics Standard 2 – Year 12

***Sample for implementation for Year 12 from Term 4, 2018***

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| **Unit title** | Modelling with Graphs | **Duration** | 20–24 hours |
| **Strand** | Algebra | **Topic** | MS-A4: Types of Relationships |
| **Subtopic focus**The principal focus of this subtopic is the graphing and interpretation of relationships, and the use of simultaneous linear equations in solving practical problems. Students develop their ability to communicate concisely, use equations to describe and solve practical problems, and use algebraic or graphical representations of relationships to predict future outcomes.Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students. | **Resources**Using graphs to solve simultaneous equations: <http://www.transum.org/software/SW/Starter_of_the_day/Students/Using_Graphs.asp>Air quality data: <http://www.environment.nsw.gov.au/aqms/hourlydata.htm>Graphing games: <http://theuniverseandmore.com/>Quadratic model <https://teacher.desmos.com/pennycircle>Population projections: <http://www.planning.nsw.gov.au/projections> and<https://www.google.com.au/publicdata/explore?ds=d5bncppjof8f9_&met_y=sp_pop_totl&idim=country:AUS:CAN&hl=en&dl=en#!ctype=l&strail=false&bcs=d&nselm=h&met_y=sp_pop_totl&scale_y=lin&ind_y=false&rdim=region&ifdim=region&tdim=true&hl=en_US&dl=en&ind=false>Population growth:<https://www.google.com.au/publicdata/explore?ds=d5bncppjof8f9_&met_y=sp_pop_grow&idim=country:AUS:USA:JPN&hl=en&dl=en>Generate information and graphs on topics of choice: <http://www.wolframalpha.com/>A unit of work on braking distance: <http://www.amsi.org.au/teacher_modules/pdfs/Maths_delivers/Braking5.pdf>Stopping distance calculator:<http://www.auburn.nsw.gov.au/Explore/RoadSafety/RoadSafetyDocuments/calculator.html> |

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| **Outcomes** | **Assessment Strategies** |
| A student:* uses detailed algebraic and graphical techniques to critically evaluate and construct arguments in a range of familiar and unfamiliar contexts MS2-12-1
* solves problems by representing the relationships between changing quantities in algebraic and graphical forms MS2-12-6
* chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times and methods for such use MS2-12-9
* uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to others and justifying a response MS2-12-10
 | Informal Assessment:At the start of the unit the teacher assesses students’ prior learning using a variety of different strategies including:* students working in small groups to brainstorm what they already know about linear graphs
* a mastery quiz
* class discussion to develop a mind map of prior learning.

During the unit the teacher may assess student progress using strategies including:* observing student engagement during in-class problem-solving tasks
* monitoring the completion of homework tasks
* reviewing samples of student work to informally assess individual progress
* providing opportunities for students to contribute to class discussion and/or group work
* posing key questions when working in one-to-one situations with a student.
* starting each lesson with a brief (5 min) quiz that reviews the key concepts of the previous lesson and key skills that will be required in the lesson that will follow.

Formal Assessment:An investigative task in which practical situations such as the number of apps downloaded from the Apple store are modelled using spreadsheets and graphing technology and analysed. |
| This unit of work builds on the Stage 6 Year 11 subtopic MS-A2 Linear Relationships. Teachers should differentiate the learning experiences to meet the needs of the students in their class. Teachers may decide that for particular groups of students they:• comprehensively review related Year 11 content before studying the Year 12 content• briefly review the related Year 11 content.Although this unit of work has been written to address the content of both A4.1: Simultaneous linear equations and A4.2: Non-linear relationships, teachers may wish to teach these as two separate units. |

| **Content** | **Teaching, learning and assessment strategies** |
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| **A4.1: Simultaneous linear equations**Students:* solve a pair of simultaneous linear equations graphically, by finding the point of intersection between two straight-line graphs, using technology **Paperclip icon**  Information and communication technology capability icon
* develop a pair of simultaneous linear equations to model a practical situation **AAM Paperclip icon** Critical and creative thinking icon  Information and communication technology capability icon
* solve practical problems that involve finding the point of intersection of two straight-line graphs, for example determine and interpret the break-even point of a simple business problem where cost and revenue are represented by linear equations **AAM Paperclip icon** Work and enterprise
 | Introductory Activity: The teacher introduces the topic by posing a problem such as: Joe and Jen are setting up a coffee cart in the city. Set-up costs were $250 and on average, it costs them $1 to produce a cup of coffee. They decide to sell coffee for $4 a cup. How many cups do they need to sell to break even?The class discusses possible ways of solving the problem which may include the strategies of guess and check, setting up a table of values or drawing a graph. The teacher may need to define the term ’break-even’. Students add this term to their personal glossary of terms.Guided Practice: The teacher reviews the skill of graphing linear equations both by hand from a table of values and using graphing software. Independent Practice: Students practise constructing the graph of linear equations and check their graphs using graphing software.The teacher defines the simultaneous solution of a pair of linear equations as a point which is common to both graphs. This definition is linked to the concept of the break-even point. Independent Practice: Students practise finding the break-even point or simultaneous solution of pairs of linear equations* given the graphs of both lines
* given one graph and then constructing the second on the same set of axes
* using graphing software to graph two linear equations.

Class Discussion: Will there always be a simultaneous solution for a pair of linear equations? Why or why not? Guided Practice: The teacher reviews the technique of developing an equation from information presented as words. For example write an equation that illustrates the relationship between:* the income generated when cupcakes are sold for $3.50 each
* the cost of hiring an electrician if there is a $80 call out fee and the charges are then $60 per half hour or part thereof
* the cost of running an incandescent light globe if the in-rush of electricity when it is initially turned on costs 1 cent and it costs 4 cents per hour to run.

Class Discussion:For the previous examples, if the relationship was graphed as a straight line what limitations could the model have? Are there any parts of the graph that have no practical meaning?Independent Practice: Students practise recognising the related quantities and selecting a variable to represent them. They then write an equation that translates the words into algebra. Guided Practice: The class returns to the introductory problem. With teacher assistance as necessary, students determine the variables in the problem, graph the linear equations they have written and solve the problem.Class Discussion:* The two lines divide the number plane into four regions. What is the practical interpretation of each of these zones?
* What is the practical meaning of the *y‑*intercept of both of the lines?

Independent Practice: Students solve practical problems using simultaneous equations. For exampleA student needs a new printer and after research, has narrowed the decision to two options. One option is to purchase a new laser printer for $275. It will print 2000 pages per cartridge and cartridges cost $28. Alternatively an inkjet printer costs $47 and prints 500 pages per cartridge and cartridges cost $30. Which option should the student buy? Justify your answer.Other applications include choosing an energy-efficient appliance or car; distance/time problems; mixing proportions of ingredients to make a blend; event organising; problems of supply and demand.Challenge:How many grams of 70% dark chocolate and 20% milk chocolate are needed to be mixed together to produce 1 kg of 40% chocolate?Issue the formal assessment task: An investigative task in which practical situations such as the number of apps downloaded from the Apple store are modelled using spreadsheets and graphing technology and analysed. Discuss the nature and scope of the task, the marking criteria and way in which the students can access feedback and support. Provide students with an opportunity to ask questions about the task and seek clarification if necessary. Clarify procedures for submission of digital files that may be included with the task. |
| **A4.2: Non-linear relationships**Students:* use an exponential model to solve problems **AAM Paperclip icon**
	+ graph and recognise an exponential function in the form $y=a^{x}$ and $y=a^{-x}(a>0)$ using technology  Information and communication technology capability icon
	+ interpret the meaning of the intercepts of an exponential graph in a variety of contexts Critical and creative thinking icon
	+ construct and analyse an exponential model to solve a practical growth or decay problem Sustainability icon
 | Introductory Activity: Students construct a simple spreadsheet that simulates the growth of a population. For example:a certain type of algae grows at a rate of 5% each day. When first observed, the algae covered 8 m2 of the surface of a pond. Draw a graph that illustrates the growth of the algae over the next two weeks. Students describe the graph and use it to:* predict the amount of algae after a specific number of days
* explore the limitations of the model
* make changes to the rate of growth and observe the changes in the graph that results.

The teacher explains the difference between a linear model and an exponential model using, for example the simple and compound interest graphs the students have encountered in the Year 11 Financial Mathematics topic. The main differences between the two types of graphs are discussed.Exploration:Using graphing software, students input the function $y=a^{x}, a>0$ and explore the effect of changing the value of $a$ on the shape of the curve. The teacher defines the term ‘asymptote’. Using a think-pair-share strategy, students generate a list of features of the graph of $y=a^{x}$ and describe the effect of changing values of $a$ on the graph.This activity is repeated for the function $y=a^{-x}, a>0$ and the differences noted.Extension: Students explore the functions $y=ka^{x}+c$ and $y=ka^{-x}+c$ for varying values of $k$ and $c$ and describe these changes in their own words. Card Sort Game:Students are given a set of cards that include graphs of a variety of different exponential functions and possible equations on separate cards. There should be more equations than graphs. Working in pairs, the students match the correct equation to its graph. Once they have eliminated the pairs of matching graph and equation, they draw the graph of each of the remaining equations. Students use graphing software to check their answers.Research Activity:Students brainstorm possible quantities that exhibit an exponential relationship. They use the internet to explore a variety of different graphs from sources such as <https://www.google.com.au/publicdata/directory> or <http://www.wolframalpha.com>. They select those which illustrate exponential relationships.Modelling Activities:Students are posed practical situations that lead to exponential models. For example:* An exponential expression such as $M=1.5(1.2)^{x}$ can be used to calculate the mass *M* kg of a baby orangutan at age  months. This model applies for a limited time, up to *x* = 6. Calculate the mass of a baby orangutan at age three months.
* <https://teacher.desmos.com/activitybuilder/custom/561d6a780784861e06c3a6d5>
* <http://www.planning.nsw.gov.au/projections>.

Students make predictions and/or recommendations using their models. The meaning of the vertical intercept in a variety of contexts is discussed. |
| * construct and analyse a quadratic model to solve practical problems involving quadratic functions or expressions of the form $y=ax^{2}+bx+c$, for example braking distance against speed **AAM Paperclip icon**  Information and communication technology capability iconLiteracy icon
	+ recognise the shape of a parabola and that it always has a turning point and an axis of symmetry
	+ graph a quadratic function using technology  Information and communication technology capability icon
	+ interpret the turning point and intercepts of a parabola in a practical context
	+ consider the range of values for $x$ and $y$ for which the quadratic model makes sense in a practical context
 | Introductory Activity:Students develop a spreadsheet of speed against stopping distance using data available at <https://www.qld.gov.au/transport/safety/road-safety/driving-safely/stopping-distances/graph/index.html> or by generating the data using a Stopping Distance Calculator available at <http://www.auburn.nsw.gov.au/Explore/RoadSafety/RoadSafetyDocuments/calculator.html>. Students graph the data using graphing software. The class discusses the shape of the graph asking ‘Is it an exponential relationship?’ and endeavour to fit an exponential function to the graph.Exploration:Using graphing software, students input the function $y=ax^{2}+bx+c$ and explore the effect of changing the values of $a, b$ and $c $on the shape of the curve. The teacher explains that such functions are called ‘quadratic’ functions and how to recognise such a function from the algebraic form. The term ‘turning point’ is defined and students describe turning points as either maximum or minimum turning points. The highest or lowest vertical value attained is defined as the maximum or minimum value. The symmetry of the graphs is explored and students recognise the axis of symmetry.Using a think-pair-share strategy, students generate a list of features of the graph of $y=ax^{2}+bx+c$ and describe the effect of changing values of $a, b$ and $c $on the shape of the graph.The class refers to the Introductory Activity and students fit a quadratic function to the graph that was generated.Research Activity:Students brainstorm possible quantities that exhibit a quadratic relationship. They use the internet to explore a variety of different graphs from sources such as <https://www.google.com.au/publicdata/directory> or <http://www.wolframalpha.com>. They select those which illustrate quadratic relationships.The meaning of the vertical and horizontal intercepts and the turning point is discussed in a variety of contexts. In each situation, the range of values for which the model has practical sense is determined.Modelling Activities:Students are posed practical situations that lead to a quadratic model. For example:* On the Earth, the equation *d =* 4.9*t*2 can be used to express the distance, *d* metres, that an object falls in *t* seconds, if air resistance is ignored. Create a table of values for the function *d =* 4*.*9*t*2 either manually or by using a spreadsheet, and use the table or an associated graph to answer questions such as: How long does it take for an object to fall 300 m?
* <https://teacher.desmos.com/pennycircle>.

Students make predictions and/or recommendations using their models. Extension: On the moon, the equation is *d =*0*.*8*t*2. What is the difference between falling distances on Earth and the moon after 5 seconds? |
| * recognise that reciprocal functions of the form $y=\frac{k}{x}$ , where $k$ is a constant, represent inverse variation, identify the rectangular hyperbolic shape of these graphs and their important features **AAM Paperclip icon**  Information and communication technology capability icon
	+ use a reciprocal model to solve practical inverse variation problems algebraically and graphically, eg the amount of pizza received when sharing a pizza between increasing numbers of people
 | Introductory Activity:A group of young people is planning a holiday and propose sharing the cost of accommodation. The property that they would like to rent costs $1500 per week and sleeps at most, 6 people. Create a table that indicates how much each person contributes if there are 2, 3, 4, 5 or 6 people. Construct a graph that illustrates the relationship.Class Discussion: The class discusses the features of the graph including its shape and asymptotes. The teacher reviews the concept of direct variation and the linear graph that results. The concept of inverse variation is introduced and the main differences between the graphs that illustrate direct and inverse variation are discussed.Exploration:Using graphing software, students input the function $y=\frac{k}{x}$ and explore the effect of changing the value of $k$ on the shape of the curve. Using a think-pair-share strategy, students generate a list of features of the graph and describe the effect of changing the value of $k$ on the graph.Extension:Repeat the exploration above for $y=\frac{k}{x+a}+c$ and describe the effect of changing the values of $k, a$ and $c $on the shape of the graph.The teacher explains that such functions are called ‘reciprocal’ functions and explains how to recognise such a function from the algebraic form. The symmetry of the graphs is explored and students recognise the axis of symmetry.The class refers to the Introductory Activity. They propose a possible algebraic relationship for the problem and refine their suggestions using graphing software.Research Activity:Students brainstorm possible quantities that could exhibit a reciprocal relationship. They use the internet to explore a variety of different graphs from sources such as <https://www.google.com.au/publicdata/directory> or <http://www.wolframalpha.com>. They select those which illustrate reciprocal relationships.In each situation, the range of values for which the model has practical sense is determined.Modelling Activities:Students are posed practical situations that lead to a reciprocal model. For example:* Crows and whelks - <https://illuminations.nctm.org/lesson.aspx?id=1560>
* Oil spills - <https://www.learner.org/workshops/algebra/workshop7/lessonplan2.html>

Students make predictions and/or recommendations using their models.  |

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| **Prior knowledge** | **Questions and prompts for Working Mathematically** | **Summary of technology opportunities** |
| Experience in: * constructing simple spreadsheets
* translating relationships expressed in words into mathematical symbols
* using graphing software to graph functions.
 | What is the same and different about …?Is … and example of …?Are there any more examples of …?Explain why …How can we be sure that …?Of what is this a special case …?Sort or organise the following according to …What else can be concluded?What if …? | Use a spreadsheet to generate tables of values and then draw the associated graphs. Use graphing software to find the point of intersection of graphs.Use the internet to obtain data that can then be graphed using graphing software.Use graphing software to create graphs of functions using a parameter and explore the effect of changing the value of parameter. |
| **Reflection on learning and evaluation – to be completed by teacher during or after teaching the unit.** |