

Mathematics Standard Year 11

Measurement Topic Guidance

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# Topic focus

*Measurement* involves the application of knowledge, understanding and skills of numbers and geometry to quantify and solve problems in practical situations.

Knowledge of measurement enables completion of daily tasks such as making time estimations, measuring medicine, finding weights and understanding areas of materials or substances.

The study of measurement is important in developing students’ ability to make reasonable estimates for quantities, apply appropriate levels of accuracy to particular situations, and apply understanding of aspects of measurement such as length, area, volume and similarity to a variety of problems.

# Prior learning

The material in this topic builds on content from the Measurement and Geometry Strand of the K–10 Mathematics syllabus, including the Stage 5.1 substrands of Area and Surface Area and Stage 5.2 substrands of Area and Surface Area and Volume.

# Terminology

|  |  |  |
| --- | --- | --- |
| absolute erroraccuracyadjacentareabase areacaloriecapacitycomposite shapecomposite solidCoordinated Universal Time energy | error hypotenuseInternational Date Linejoulelimits of accuracylatitudelongitudemassmetric prefixesnutrition opposite | percentage errorperimeterposition coordinatesPythagoras’ theoremsector of circlesignificant figuresstandard formtime zoneTrapezoidal rule ****volume |

# Use of technology

Students should be given the opportunity to use suitable software applications to draw accurate figures, investigate geometrical properties, and create both two-dimensional (2D) and three-dimensional (3D) drawings. A spreadsheet could be used to investigate maximising the perimeters and areas of shapes.

Online conversion calculators are readily available and may be used to investigate conversion of units.

Online maps (eg as available through Google Earth) may be used to enhance visualisation of scale, to calculate local distances, and to make estimations of travelling times. Such resources can also be used to make estimations and calculations of areas of land using field diagrams.

# Background information

Some students may be interested in information related to the history of the development of latitude and longitude, time measurement and the International Date Line (IDL). Various resources such as DVDs and the internet are available to support independent research.

The ancient Phoenician maritime traders located their position on the globe using astronomical observations and are attributed as the first to determine latitude. The Greek mathematicians, Eratosthenes and Hipparchus made improvements, using trigonometry to develop a grid system for locating position on the globe. It was not until the 1760s when the English inventor John Harrison determined longitude through precise time-measurement that longitude was accurately determined.

Coordinated Universal Time (UTC) now replaces Greenwich Mean Time (GMT) as the time standard by which the world’s time is regulated.

# General comments

Where possible, students should be given the opportunity to gain practical experiences in relation to the mathematics addressed in this topic area and relevant use of technology. Where it is not possible to provide practical experiences, the problems posed should be relevant to the lives of students.

This topic provides many opportunities for students to acquire and apply skills through extended real-life investigations. Interest areas could include, for example, resource usage, food and nutrition, the human body, travel and sustainability.

# Future study

Students undertaking the Year 12 Mathematics Standard course will apply their knowledge of units of measurement to solving problems involving ratios, rates and scale drawings. In Mathematics Standard 2, students will also encounter units of measurement in Non-right-angled Trigonometry.

Knowledge of measurement will provide students with many of the key skills needed when they undertake further studies or enter the workforce.

# Subtopics

* MS-M1: Applications of Measurement 
* MS-M2: Working with Time

## MS-M1: Applications of Measurement Paperclip icon

### Subtopic focus

The principal focus of this subtopic is to develop an awareness of the inherent error in measurements and to become competent in solving practical problems involving energy, mass, perimeter, area, volume and capacity.

Students develop knowledge of the concepts of measurement and demonstrate fluency with its application.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

## M1.1 Practicalities of measurement

### Considerations and teaching strategies

* Teachers may need to revise with students:
* determining the appropriate units to use when measuring
* conversion between commonly used units using standard prefixes
* accuracy of physical measurement being limited to $\pm \frac{1}{2}$ of the smallest unit of which the measuring instrument is capable, including determination of possible sources of error in measuring.
* Reduce the likelihood of error by repeating and averaging measurements.
* Investigate the degree of accuracy of reported measurements, including the use of significant figures where appropriate.
* Standard form is also referred to as standard index form or scientific notation.
* Use standard form and standard prefixes in the context of measurement
* standard prefixes include nano-, micro-, milli-, centi-, kilo-, mega-, giga- and tera-
* express measurements in standard form.
* Convert between common units for length, area, volume and capacity.
* In the context of measurement, error does not indicate a mistake, it is the term used to refer to the precision of a measurement.

### Suggested applications and exemplar questions

* Students may calculate the percentage error in a measurement, for example if the measured height of a wall was 155 cm $\pm $ 0.5 cm (ie to the nearest centimetre), the percentage error for this measurement is $\left(\frac{0.5}{155}\right)×100\%$.
* Students may measure their heights and calculate the percentage error in the measurement.

## M1.2: Perimeter, area and volume

### Considerations and teaching strategies

* Teachers may need to revise with students calculations involving:
* perimeter, area and surface area, volume and capacity
* Pythagoras’ theorem.
* Students should be extended to calculate:
* the areas of:
	+ composite shapes/figures
	+ annuluses and parts of a circle (eg quadrant, sector).
* the surface area of:
	+ prisms and pyramids
	+ cylinders (without ‘top’ and/or ‘bottom’) and closed cylinders
	+ spheres.
* the volume of:
	+ cones
	+ square and rectangular pyramids
	+ right prisms, where the base is a composite or irregular two-dimensional shape, eg an I-beam
	+ composite solids
	+ an annular cylinder.
* Manipulation of nets may be used to enhance understanding of the surface area of prisms, cylinders and pyramids.
* Students could apply algebraic skills to find a missing dimension of
* a shape given its perimeter or area
* a solid given its area, surface area, volume or capacity.
* Students could apply geometric skills to find a missing dimension of a shape in order to find its perimeter, area or volume. This may involve applying the scale factor to find an unknown side in similar triangles.
* Data is widely available on the internet. For example, useful information and data regarding rainfall can be found at the [Australian Bureau of Meteorology](http://www.bom.gov.au/) and [Sydney Water](http://www.sydneywater.com.au/SW/index.htm) websites.
* The cost of water may vary depending upon locality. Investigations of cost are not intended to be limited to town water only. Teachers may investigate relevant local methods of water accessibility and cost. Comparisons between the cost of water in cities and country areas can be made.
* The catchment area of a roof is the ‘plan view area’ of the roof and not the actual area of the roofing material. It should be noted that the roof catchment area for single-storey houses is usually greater than the floor area if the house has eaves.

### Suggested applications and exemplar questions

Students could:

* study aspects of water as a resource, including rainfall, personal water usage, and local, state and national water usage, and compare rainfall and water usage in a range of countries. They could explore, compare and solve problems based on the water availability and usage. For example:
* interpret information about a household’s water usage from a water bill
* investigate household water usage in different Australian and international locations
* construct and interpret rainfall tables of data and graphs.
* determine errors in calculations resulting from errors made in measurement.
* calculate the volume of water held by tanks of various shapes and sizes.
* calculate the volume of water that can be collected on a roof given the rainfall in millimetres and the roof area (plan view area) and determine the most suitable size tank for a given roof.
* use the following rule-of-thumb approach to estimating the collection capacity of a roof: every 1 millimeter (mm) of rain = 1 litre (L) of water per square metre (m2) of roof area, and then allow for a 15% wastage factor.
* compare the amount of water used by a household and the amount of rainfall that could be collected over a given period.
* investigate the costs of water usage at local, state and national levels, using published data.
* determine the amount and cost of water used by various household activities, including showering and bathing, washing clothes, watering the garden, washing a car, and using the toilet.
* log their water use over a one-week period and use this to estimate personal water usage and costs over longer time periods.
* perform calculations to compare the amount of water used by products with various ratings, eg dishwashers and washing machines.
* measure and divide an irregularly shaped block of land into regular shapes and use this information to calculate its approximate area.
* determine the number of bricks in 1 square metre of a brick wall.
* discuss and report on possible sources of error, eg experimental, instrumental and constant error when determining the number of bricks in one square metre of a brick wall.
* estimate the painted surface area of a classroom and determine how much paint needs to be purchased for a particular coverage rate, eg 1 litre covers 14 m2 .
* investigate the dimensions that maximise the area for a given shape and perimeter, eg in the design of playpens and stock paddocks.
* use practical methods to investigate units of volume and capacity, eg by pouring contents of a soft drink bottle into a rectangular prism.
* calculate how many kilograms of chicken manure would be required to fertilise a football field 100 m $×$ 50 m if it is required that 8 g/m2 be applied.
* design the shape and dimensions of a container that would have a particular capacity, given the purpose and use of the container.
* work in small groups to design cost-effective packaging, eg given four table-tennis balls, design two different boxes to package them and determine the better of the two designs in terms of minimisation of material used.
* design an open rectangular water tank given a square piece of sheet metal 2 m $×$ 2 m
* the volume of water that the tank will hold depends on the size of squares cut from each of the four corners of the piece of metal
* choose a scale, make models and find the volume of water that can be held
* graph results and determine when the volume is the greatest
* consider what happens if the side of the original square is doubled.
* calculate the surface area and volume of sphere-shaped sweets or cylinder-shaped sweets, eg marshmallows.
* Class results could be combined to form a data set for statistical analysis.
* Discussion could include:
	+ What assumptions are being made about the shape of the sweet?
	+ How can a reasonably accurate measure for the radius be obtained?
	+ How accurate will the results be?
	+ Are the results normally distributed?
	+ What claims could the manufacturer make about the surface area and volume of that particular product?
* use the Trapezoidal rule to find the area of an irregular shape. For example,

The diagram below represents a field.



Use the Trapezoidal rule to find the area of the field.

*Solution:*

Method 1 $A≈\frac{3}{2}\left(5+7\right)+\frac{3}{2}\left(7+12\right)+\frac{3}{2}\left(12+8\right)+\frac{3}{2}\left(8+10\right)$

 $A≈103.5$

 $∴$ the area is approximately 103.5 m2

 Method 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$x$$ | 0 | 3 | 6 | 9 | 12 |
| height | 5 | 7 | 12 | 8 | 10 |
| weight | 1 | 2 | 2 | 2 | 1 |

$$A≈\frac{3}{2}\left(5+2×7+2×12+2×8+10\right)$$

 $A≈103.5$

 $∴$ the area is approximately 103.5 m2

## M1.3 Units of energy and mass

### Considerations and teaching strategies

* The watt is the International System of Units (SI) derived unit of power and is equal to one joule per second. By definition, power is a rate. The symbol for the watt is W.
* Students should be aware of energy rating labels and be able to interpret the symbols and figures.
* The energy consumptions of common appliances and physical activities can be ranked by students. Examples that could be used include:
* a typical household incandescent light bulb uses 40 W to 100 W
* a person climbing a flight of stairs is doing work at the rate of approximately 200 W
* a highly-trained athlete can work at up to approximately 2000 W for brief periods
* a car engine produces approximately 25 000 W while cruising.
* One watt-hour is the amount of energy (usually electrical) expended by a one-watt load (eg a light bulb) drawing power for one hour. The watt-hour (symbol W·h or Wh) is a unit of energy. It is most commonly used on household electricity meters in the form of the kilowatt-hour (kW·h or kWh), which is 1000 watt-hours.
* Quantities and units may be expressed in both decimal form and standard form,

eg 6.8 $×$ 103 MW or 6 800 000 kW.

* Conversion of units is to be based on the table of units below:

|  |  |  |
| --- | --- | --- |
| Multiple | Name | Symbol |
| 10–3 | milliwatt | mW |
| 100 | watt | W |
| 103 | kilowatt | kW |
| 106 | megawatt | MW |
| 109 | gigawatt | GW |

* Although the kilojoule (kJ) is the Australian measure of the energy value of food or drink, many nutritional information panels on Australian food labels also include information about what we call ‘calories’ (Cal) but which are actually kilocalories (kcal). This can be confusing and so the following conversions are provided:
	+ 1 cal $≈$ 4.186 8 j
	+ 1000 cal $=$ 1 kcal $=$ 1 Cal $≈$ 4.186 8 kJ
* When solving problems involving the conversion of these units, students should be provided with the conversion factor to a reasonable level of accuracy.
* Further information about Australian standards of measurement and conversion factors is available online in the [Australian National Measurement Regulations 1999](https://www.legislation.gov.au/Details/F2015C00296). Australian standards concerning nutrition information panels can be found at [Food Standards Australia and New Zealand.](http://www.foodstandards.gov.au/consumer/labelling/panels/Pages/default.aspx)
* Data in relation to worldwide electricity consumption is available on the internet.

### Suggested applications and exemplar questions

Students could:

* explore, compare and solve problems based on the consumption of household electricity. For example:
* interpret information about a household’s electricity usage, eg a household electricity bill
* rank common appliances and physical activities in terms of their energy consumption in watts
* describe the watt-hour and kilowatt-hour as units of energy usage
* calculate the cost of running different household appliances for various time periods, given the power rating, usage time, and cost of power
* perform calculations and conversions involving units related to power, eg watt, watt-hour
* interpret the energy rating of appliances and compare running costs of different models of the same type of appliance
* calculate the amount of electricity that could be saved by using energy-efficient devices and adopting energy-efficient strategies.
* calculate the running costs of various appliances, eg
1. calculate the cost of running a 200-watt television set for six hours if the average peak rate for domestic electricity is $0.15/kWh.

*Solution*

Total electricity used = 200 × 6 = 1200 watt-hours or 1.2 kWh.

Cost of electricity used is 1.2 kWh × $0.15/kWh = $0.18.

1. calculate the cost of running a 2400-watt (2.4 kW) fan heater for eight hours per day for 30 days. Assume electricity is charged at $0.18/kWh.

*Solution*

Total electricity used = 2.4 × 8 × 30 = 576 kWh.

Cost of electricity used is 576 kWh × $0.18/kWh = $103.68.

* extract data from tables or graphs and interpret the information in order to compare electricity consumption in different countries.
* calculate the mass of a solid after finding its volume, eg A machine piece called a cam is in the form of a truncated cone as illustrated in the diagram.

6 cm

9 cm

$x$ cm

21 cm

NOT TO

SCALE

1. Use similar triangles to find the dimension labelled $x$.
2. Find the volume of the cam.
3. It is known that the mass of steel is 7 850 kg/m3. Use this information to find the mass of the cam correct to the nearest gram.

## MS-M2: Working with Time

### Subtopic focus

The principal focus of this subtopic is to understand concepts related to locations on Earth’s surface and calculation of distances and time differences using latitude, longitude and time zones.

Students develop awareness of being a global citizen and the relationships between different countries in terms of location, distance and time.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

### Considerations and teaching strategies

* Use a physical globe or sphere to help students distinguish between great and small circles.
* Discuss the use of the equator and the Greenwich Meridian as lines of reference for locations on the Earth’s surface.
* Locate positions on the surface of the Earth using latitude and longitude.
* Use the internet to explore the history of time zones: <http://www.bbc.co.uk/news/world-12849630>
* Use time zones and the International Date Line in solving problems.
* Plot longitude on a number line to provide a visual stimulus when calculating time differences. From a number line, students can readily determine whether to add or subtract time.

### Suggested applications and exemplar questions

Students could:

* calculate the present time in all the capital cities of Australia.
* investigate whether or not each of the states and territories in Australia has daylight saving and, if so, when it starts and finishes.
* find three cities with the same time, for both a given number of hours earlier and later than local time
* these could be marked on a world map and displayed to reinforce the concept of time zones.
* Simulate an ‘Amazing Race’ game where students are given a series of destinations in terms of their latitude and longitude. For example,
* Using a map or the internet, determine the country and name of the city located at the destination.
* Determine the shortest route that ensures all destinations are visited.
* Estimate the time taken to travel between each destination.
* Given a location and time at which a conference call will be made including people at all destinations, determine the time at which each call needs to be placed.